Hausdorff dimension of the diagram
of Hölder continuous functions
L. Biacino

Abstract

In this paper the relationship between the set E of the α-essential points of an α-Hölder continuous function $f : [a, b] \to R$ ($0 < \alpha < 1$) and the Hausdorff dimension of the graph G of f (in brief H-dim$G$) is investigated. It is well known that H-dim$G \leq 2$; moreover in [2] a sufficient condition about the set E in order that H-dim$G = 2 - \alpha$ was determined. Here we determine some necessary condition about the set E if H-dim$G = 2 - \alpha$. Among other things we prove that E is a nonempty closed set and if the α-Hölder continuous function f is also μ-Hölder continuous, with $\mu > \alpha$, in every closed interval enclosed in $[a, b] - E$, then $D^r(E)$ is nonempty for every $r \in N$, $D^r(E)$ being the Cantor r-th derivate set of E.

[2]L. Biacino: "Derivatives of fractional order of continuous functions", Ricerche di Matematica 53 (2004), 231–254